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CHAP: 5: OSCILLATIONS

* Oscillatory Motion: It is the motion in which the body repeatedly moves to & fro about the mean position along the same path.

eg. 1) The motion of a pendulum in a clock.
2) The motion of the prongs of a vibrating tuning fork.
3) The oscillations of spring.

* Periodic Motion: It is the motion in which a body repeats itself in equal interval of time (along certain path)

eg. 1) The motion of minute hand of a clock.
2) The motion of the moon around the earth.
3) The motion of electrons around the nucleus.

* Every oscillatory motion is periodic but every periodic motion need not be oscillatory.

eg. circular motion is periodic but it is not oscillatory.

* Simple form of oscillatory periodic motion is the simple harmonic motion (S.H.M.)

• If the path of S.H.M. is straight line - linear SHM.
• If the path of SHM is an arc of circle - Angular SHM.

Que - Define linear SHM; Explain it by spring-mass oscillator.

→ Linear SHM:- It is defined as the linear periodic motion of a body, in which force (acceleration) acting on a body is always directed towards the mean position and its magnitude is directly proportional to the displacement from the mean position.

Consider a block of mass 'm', held on a frictionless horizontal surface with the help of spring. One end of spring is fixed at rigit support. & at the other end a block is fixed.

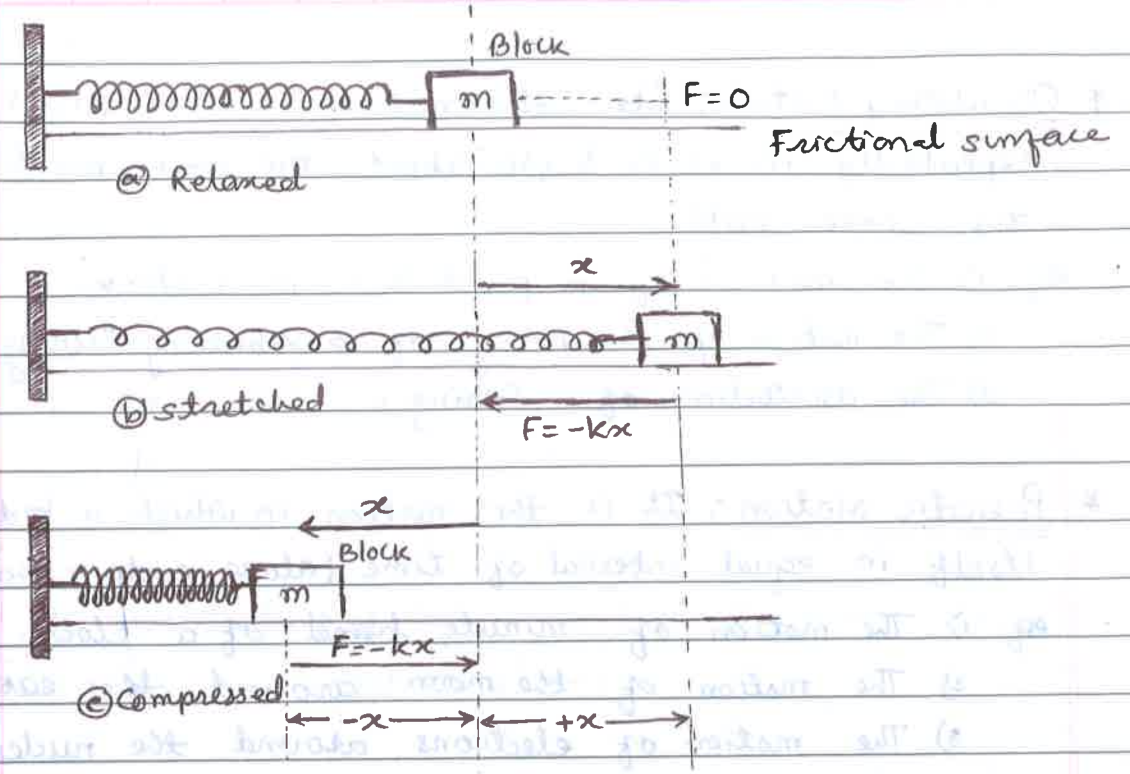


Fig (a) shows the equilibrium position in which the spring exerts no force on the block; the spring is relaxed.

As shown in fig (b) pull the block of mass m towards the right & release it, the block will begin its to & fro motion on either side of its equilibrium position. in fig (b) the force exerted by the spring on the block is directed towards the left; due to its elastic properties.

As shown in fig (c) the block reached to extreme position towards left, through distance x, due to elasticity the spring exerts force on the block towards mean position.

If x is the displacement, the restoring force f is given by

$f = -kx$ —(1) -- k is constant called as force constant

(-ve sign is due to f & x are opposite).

By Newtons 2nd law

$f = ma$

$\therefore a = \frac{f}{m}$ —(2)

put (1) in (2)

$\therefore a = -\left(\frac{k}{m}\right)x$ —(3)

but $a = -\omega^2 x$

$\therefore \omega = \sqrt{\frac{k}{m}}$ —(4)

Eqⁿ (4) gives angular freqⁿ of oscillation.

Que. Find the differential equation of linear S.H.M.

⟹ In a linear SHM, the restoring force acting on a body is

$f = -kx$ —(1)

By Newtons 2nd law

$f = ma$ —(2)

From eqⁿ (1) & (2)

$ma = -kx$ —(3)

but $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$\therefore m\frac{d^2x}{dt^2} = -kx$

$\therefore m\frac{d^2x}{dt^2} + kx = 0$ -- Divide by m

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$ —(4)

but $\frac{k}{m} = \omega^2$ ---- (ω = angular freqⁿ)

$\therefore \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$ —(5) Eqⁿ (5) gives differential eqⁿ of linear SHM.

Que. state the differential equation of linear SHM. & find the expressions of acceleration, velocity & displacement of SHM.

⟹ The differential eqⁿ of linear SHM is

$\frac{d^2x}{dt^2} + \omega^2 x = 0$ —(1)

I) Accⁿ of SHM

From eqⁿ (1)

$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x}$ —(2) — Eqⁿ (2) gives accⁿ of particle in SHM. ($\because a = -\omega^2 x$)

II) Velocity in SHM - From eqⁿ (1)

but $\frac{d^2x}{dt^2} = \frac{d}{dt}\cdot\frac{dx}{dt} = \frac{dv}{dt} \times \frac{dx}{dx} = \frac{dx}{dt}\cdot\frac{dv}{dx} = v\cdot\frac{dv}{dx}$

$\therefore v\frac{dv}{dx} = -\omega^2 x$

$v\,dv = -\omega^2 x\,dx$ —(3)

Integrating both sides.

$$\int v\,dv = -\omega^2 \int x\,dx$$

$$\frac{v^2}{2} = -\frac{\omega^2 x^2}{2} + C \quad \text{--- (4)}$$

where C is constant of integration.

Let A be the max^m displacement (amplitude) of SHM.

extreme	mean	extreme
$x=-A$	$x=0$	$x=+A$
$V=0$	$V=\max^m$	$V=0$

When the particle is at the extreme position $x=A$ & $V=0$.

$\therefore$ From eq^n (4)

$$0 = -\frac{\omega^2 A^2}{2} + C$$

$$\therefore C = \frac{\omega^2 A^2}{2} \quad \text{--- (5)}$$

put (5) into (4)

$$\frac{v^2}{2} = \frac{-\omega^2 x^2}{2} + \frac{\omega^2 A^2}{2}$$

$$\therefore v^2 = \omega^2 (A^2 - x^2)$$

$$\therefore \boxed{v = \pm \omega \sqrt{A^2 - x^2}} \quad \text{--- (6)}$$

Eq^n (6) gives velocity of particle in SHM.

III) Displacement in SHM :-

put $v = \frac{dx}{dt}$ in eq^n (6)

$$\therefore \frac{dx}{dt} = \omega \sqrt{A^2 - x^2}$$

$$\therefore \frac{dx}{\sqrt{A^2 - x^2}} = \omega dt \quad \text{--- (7)}$$

Integrating both sides

$$\int \frac{dx}{\sqrt{A^2 - x^2}} = \int \omega dt$$

$$\therefore \sin^{-1}\left(\frac{x}{A}\right) = \omega t + \phi .$$

$$\therefore \frac{x}{A} = \sin(\omega t + \phi)$$

$$\therefore \boxed{x = A\sin(\omega t + \phi)} \quad \text{--- (8)}$$

Eq^n (8) gives displacement of particle in SHM.

Case I :- If the particle starts SHM from the mean position

At mean position $x=0$, $t=0$

From eq^n (8)

$A\sin\phi = 0 \qquad \because A \neq 0$

$\therefore \sin\phi = 0$

$\therefore \phi = 0$ or $\phi = \pi$.

$\therefore$ From eq^n (8)

$$\boxed{x = \pm A\sin\omega t} \quad \text{--- (9)}$$

Case II : If the particle starts SHM from the extreme position.

At extreme $x = \pm A$, at $t=0$

From eq^n (8)

$\pm A = A\sin\phi$.

$\therefore \sin\phi = \pm 1$

$\therefore \phi = \frac{\pi}{2}$ or $\phi = \frac{3\pi}{2}$.

$\therefore$ From eq^n (8)

$x = A\sin(\omega t + \frac{\pi}{2})$ or $x = A\sin(\omega t + \frac{3\pi}{2})$

$$\therefore \boxed{x = \pm A\cos\omega t} \quad \text{--- (10)}$$

* Max^m & Min^m values

1) Displacement in SHM :

The general eq^n of displacement is

$x = A\sin(\omega t + \phi)$.

At mean position $(\omega t + \phi) = 0$ or π.

$\therefore x_{min} = 0$

ie displacement is min^m at mean position.

At extreme position $(\omega t+\phi) = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

$\therefore x = A\sin\frac{\pi}{2} = A\times 1 = +A$

$x = A\sin\frac{3\pi}{2} = A\times(-1) = -A$

$\therefore x_{max} = \pm 1$

The displacement is max^m at extreme position.

2) **velocity in SHM:**

The eq^n of velocity in SHM is

$V = \pm\omega\sqrt{A^2-x^2}$.

At mean position $x=0$

$\therefore V_{max} = \pm A\omega$

The velocity is max^m at mean position

At extreme position $x=\pm A$

$\therefore V_{min} = 0$

The velocity is min^m at extreme position.

3) **Acceleration in SHM:**

The magnitude of acc^n in SHM is

$a = \omega^2 x$.

At mean position $x=0$

$\therefore a_{min} = 0$

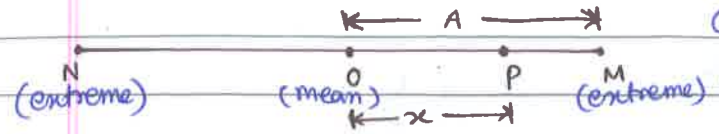
The acc^n is min^m at mean position.

At extreme position $x=\pm A$.

$\therefore a_{max} = \pm\omega^2 A$

The acc^n is max^m at extreme position.

* **Amplitude of SHM (A)** It is the maximum displacement of a particle performing SHM from its mean position.



Consider a particle P performing SHM along a straight line MN. 'O' is the centre of line MN.

The displacement of SHM is

$x = A\sin(\omega t+\phi)$ —— ①

The particle will have its max^m displacement when $\sin(\omega t+\phi)=\pm 1$

ie $\underline{x=\pm A}$, The distance A is called the amplitude of SHM.

* **Period of SHM (T):** It is the time taken by the particle performing SHM to complete one oscillation.

The displacement of the particle at time 't' is

$x = A\sin(\omega t+\phi)$ —— ①

After time $t = (t+\frac{2\pi}{\omega})$, the displacement will be

$x = A\sin\left[\omega\left(t+\frac{2\pi}{\omega}\right)+\phi\right]$

$\therefore x = A\sin(\omega t+2\pi+\phi)$

$\therefore x = A\sin(\omega t+\phi)$ —— ②

From eq^n ① & ②, it shows that, the particle is at the same position after a time $\frac{2\pi}{\omega}$.

$\therefore \boxed{T = \frac{2\pi}{\omega}}$ ③ It is the min^m time after which it repeats.

As $\omega^2 = \frac{k}{m} = \frac{\text{force per unit displacement}}{\text{mass}} =$ Acc^n per unit displacement.

$$T = \frac{2\pi}{\sqrt{\text{Acceleration per unit displacement}}}$$

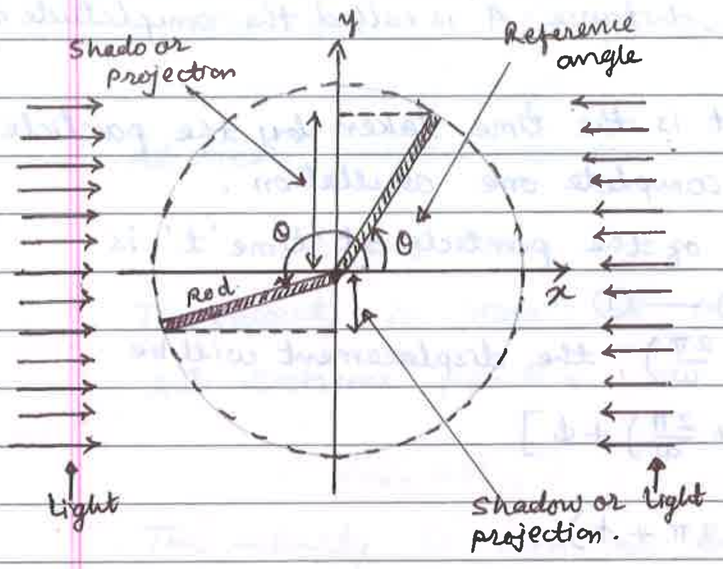
$$\therefore \boxed{T = 2\pi\sqrt{\frac{m}{k}}} \quad ④$$

* **Frequency of SHM (n):** The number of oscillations performed by a particle performing S.H.M. per unit time is called the freq^n of SHM.

$$n = \frac{1}{T} = \frac{\omega}{2\pi} \quad --- \left(\omega^2 = \frac{k}{m}\right)$$

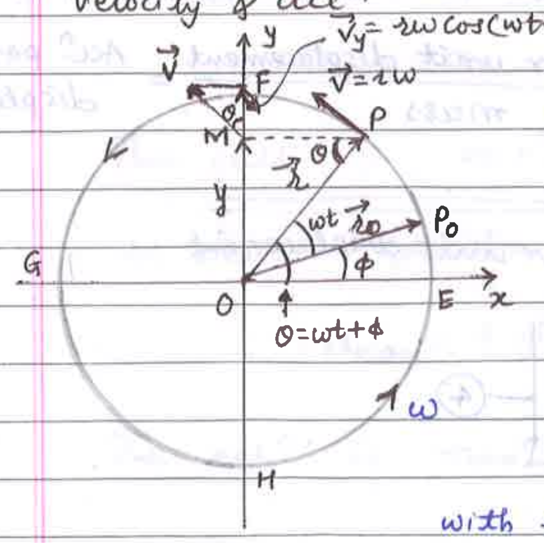
$$\therefore \boxed{n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}}$$

* **Projection of rotating rod:-** As shown in fig a rod rotating along a vertical circle in xy-plane. The rod is illuminated parallel to x-axis from either side by linear source parallel to the rod. The shadow (projection) of the rod will be produced on the y-axis. The tip of this shadow can be seen to be oscillating about the origin along the y-axis.



It shows that the tip of rod performs UCM & its projection performs an SHM corresponding to its motion.

Que. Describe SHM as a projection of a UCM for displacement, velocity & acc^n.



A particle 'p' performs UCM with centre at O. Its angular positions are decided with reference OX. If particle at E, its angular position is zero. at F it is $90° = \frac{\pi}{2}^c$. at G it is $180° = \pi^c$ & so on. If it comes to E, it will be $360° = 2\pi^c$.

At $t=0$, the particle is at P_0, with reference angle (initial phase) ϕ. In time t, its angular displacement is ωt.

So the reference angle is $\theta = \omega t + \phi$. Let FH is the diameter along y-axis & label OM as the projection of $\vec{r} = OP$ on this.

1) **Projection of displacement:-**

At time 't', the projection of particle is

$OM = OP \sin\theta$

$\therefore \boxed{y = r\sin(\omega t + \phi)}$ — ①

This is the eq^n of linear SHM of amplitude 'r', ω is angular velocity (angular freq^n), $\theta = (\omega t + \phi)$ is phase of SHM.

2) **Projection of velocity:**

The instantaneous velocity of the particle P in the circular motion is tangential velocity of magnitude $r\omega$. Its projection on reference diameter FH, is given by

$V_y = r\omega\cos\theta$

$\therefore \boxed{V_y = r\omega\cos(\omega t + \phi)}$ → ② This is the eq^n of velocity of particle performing SHM.

3) **Projection of acceleration:**

The instantaneous acc^n of particle (p) in circular motion is the radial (centripetal) acc^n of magnitude $r\omega^2$. Its projection on the reference diameter FH is

$a_y = -r\omega^2\sin\theta$

$\therefore a_y = -r\omega^2\sin(\omega t + \phi)$ — ③ From eq^n ① & ③.

$\therefore \boxed{a_y = -\omega^2 y}$ — ④ This the eq^n of acc^n of particle performing SHM.

Que^n - Describe graphical representation of displacement, velocity & acc^n of SHM w.r.t. time (or phase).

(a) When the particle starting from mean position.

(b) When the particle starting from extreme position.

State the conclusions from the graph.

→ (a) **When the particle starting from mean position:-**

As the particle starts from mean position, the displacement of SHM is

$x = A\sin\omega t$ — ①

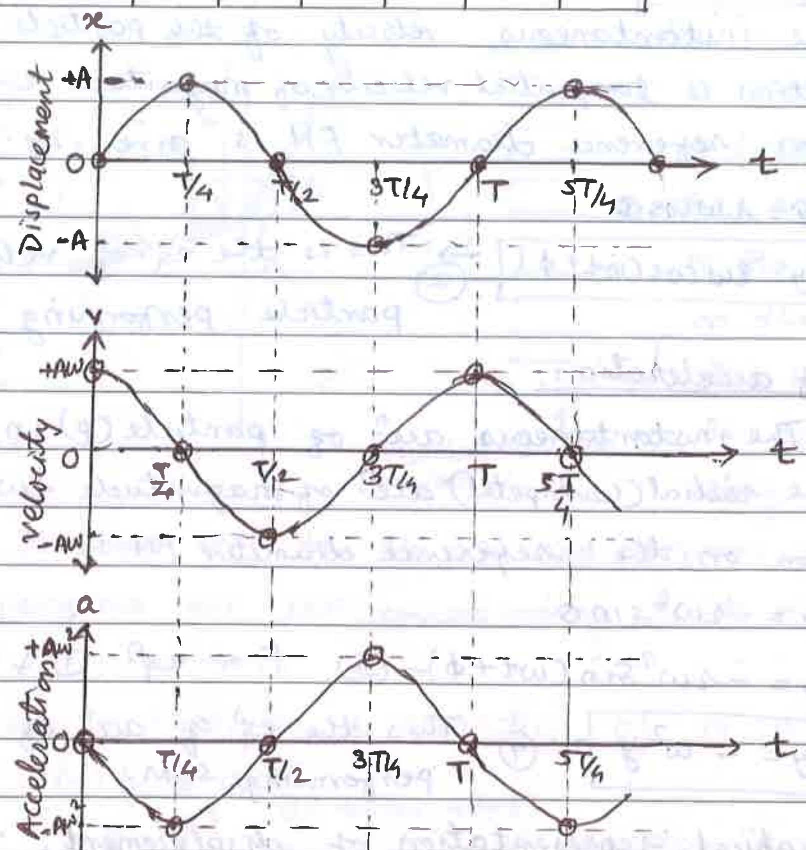
The velocity of SHM is $v = \frac{dx}{dt}$

($\therefore V = A\omega\cos\omega t$ — ②

The acc^n of SHM is $a = \frac{dv}{dt}$

$\therefore a = -A\omega^2 \sin \omega t$ — ③

Time (t)	0	T/4	T/2	3T/4	T	5T/4
phase (θ)	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$
x	0	+A	0	−A	0	+A
V	+Aω	0	−Aω	0	+Aω	0
a	0	$-A\omega^2$	0	$+A\omega^2$	0	$-A\omega^2$



✗ **Conclusions:-**

1) Displacement, velocity and acceleration of SHM are periodic functions of time.
2) Displacement time & acc^n time curves are sine curves & velocity time curve is cosine curve.
3) The phase diff. bet^n displacement and velocity is $\pi/2^c$
4) The phase diff bet^n velocity and acceleration is $\pi/2^c$.
5) The phase diff bet^n displacement and acceleration is π^c.
6) Shapes of all the curves repeated after $2\pi^c$ or after a time T.

* Point ⓑ when particle starts from extreme position. (H.W.)

Que: Describe the composition of two SHMs having same period and parallel to each other (along same path), Find the eq^n of their resultent amplitude. state the special cases for phase diff 0^c, $\pi/2^c$ & π^c. Also find the phase of resultent SHM.

⇒ Consider two SHMs having same period $(T = \frac{2\pi}{\omega})$, along same path (x-axis) A_1 & A_2 be their amplitudes. ϕ_1 & ϕ_2 be their initial phases, which are

$x_1 = A_1 \sin(\omega t + \phi_1)$ — ①

$x_2 = A_2 \sin(\omega t + \phi_2)$ — ②

The resultent S.H.M. is

$x = x_1 + x_2$ — ③

put ① & ② into ③

$x = A_1 \sin(\omega t + \phi_1) + A_2 \sin(\omega t + \phi_2)$ -- $[\sin(A+B) = \sin A \cos B + \cos A \sin B]$

$x = A_1 \sin \omega t \cos \phi_1 + A_1 \cos \omega t \sin \phi_1$
$+ A_2 \sin \omega t \cos \phi_2 + A_2 \cos \omega t \sin \phi_2$

$x = [A_1 \cos \phi_1 + A_2 \cos \phi_2] \sin \omega t + [A_1 \sin \phi_1 + A_2 \sin \phi_2] \cdot \cos \omega t$ — ④

Let $R \sin \delta = A_1 \sin \phi_1 + A_2 \sin \phi_2$ — ⓐ

$R \cos \delta = A_1 \cos \phi_1 + A_2 \cos \phi_2$ — ⓑ

put ⓐ & ⓑ into ④

$x = R \cos \delta \cdot \sin \omega t + R \cos \omega t \sin \delta$

$x = R[\sin \omega t \cos \delta + \cos \omega t \sin \delta]$

$\therefore \boxed{x = R \sin(\omega t + \delta)}$ — ⑤

Eq^n ⑤ is the resultent S.H.M. of amplitude R & same period $(T = \frac{2\pi}{\omega})$

* **Amplitude of resultent SHM (R):**

squaring and adding eq^{ns} ⓐ & ⓑ

$R^2 \sin^2 \delta + R^2 \cos^2 \delta = (A_1 \sin \phi_1 + A_2 \sin \phi_2)^2 + (A_1 \cos \phi_1 + A_2 \cos \phi_2)^2$

$R^2 = A_1^2 \sin^2 \phi_1 + A_2^2 \sin^2 \phi_2 + 2A_1A_2 \sin \phi_1 \cdot \sin \phi_2$
$+ A_1^2 \cos^2 \phi_1 + A_2^2 \cos^2 \phi_2 + 2A_1A_2 \cos \phi_1 \cdot \cos \phi_2$

$R^2 = A_1^2 + A_2^2 + 2A_1A_2[\sin \phi_1 \sin \phi_2 + \cos \phi_1 \cdot \cos \phi_2]$

$R^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_1 - \phi_2)$

$\therefore \boxed{R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_1 - \phi_2)}}$ — ⑥

Special cases

I] When phase diff is $(\phi_1-\phi_2=0)$, the two SHMs are in phase

$\because \phi_1-\phi_2=0$, $\cos(\phi_1-\phi_2)=\cos 0=1$

From eqⁿ ⑥

$$\therefore R=\sqrt{A_1^2+A_2^2+2A_1A_2}$$

$$R=\sqrt{(A_1+A_2)^2}$$

If $A_1=A_2=A$ $\therefore R=2A$

$$\therefore R=\pm(A_1+A_2)$$

II] when phase diff is $(\phi_1-\phi_2=\pi/2)$, the two SHMs are out of phase

$\because \phi_1-\phi_2=90^\circ$, $\therefore \cos(\phi_1-\phi_2)=\cos 90^\circ=0$

$\therefore$ From eqⁿ ⑥

$$R=\sqrt{A_1^2+A_2^2}$$

If $A_1=A_2=A$ $\therefore R=\sqrt{2}\,A$

III] when phase diff is $(\phi_1-\phi_2=\pi)$, the two SHMs are out of phase

$\because \phi_1-\phi_2=180^\circ$, $\therefore \cos(\phi_1-\phi_2)=\cos 180^\circ=-1$

$\therefore$ From eqⁿ ⑥

$$R=\sqrt{A_1^2+A_2^2-2A_1A_2}$$

$$R=\sqrt{(A_1-A_2)^2}$$

If $A_1=A_2=A$ $\therefore R=0$

$$\therefore R=|A_1-A_2|$$

* Initial phase of resultant SHM (δ):

Divide eqⁿ ⓐ by ⓑ

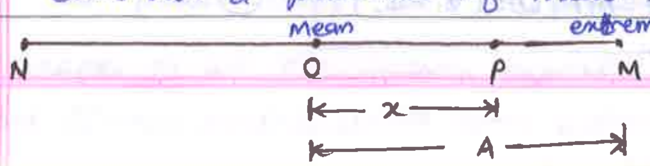
$$\frac{R\sin\delta}{R\cos\delta}=\frac{A_1\sin\phi_1+A_2\sin\phi_2}{A_1\cos\phi_1+A_2\cos\phi_2}$$

$$\tan\delta=\frac{A_1\sin\phi_1+A_2\sin\phi_2}{A_1\cos\phi_1+A_2\cos\phi_2}$$

$$\therefore \boxed{\delta=\tan^{-1}\left[\frac{A_1\sin\phi_1+A_2\sin\phi_2}{A_1\cos\phi_1+A_2\cos\phi_2}\right]} \quad \text{—⑦}$$

Que- Deduce the expressions of KE & PE of SHM. Hence find TE of SHM. state the diff. cases.

⟹ consider a particle of mass 'm' performing linear SHM along the path MN about mean position 'O'. 'P' be a point



at distance 'x' from mean position.

* KE of SHM:

The velocity of SHM is

$$V=\omega\sqrt{A^2-x^2} \quad \text{—①} \quad \text{---}(A=\text{Amplitude},\ x=\text{Displacement}).$$

The KE of SHM is

$$KE=\tfrac{1}{2}mv^2 \quad \text{—②}$$

$$\therefore \boxed{KE=\tfrac{1}{2}m\omega^2(A^2-x^2)} \quad \text{—③} \qquad \because \omega^2=\frac{k}{m} \Rightarrow k=m\omega^2.$$

$$\therefore \boxed{KE=\tfrac{1}{2}k(A^2-x^2)} \quad \text{---}(KE=E_K)$$

Also $\because x=A\sin(\omega t+\phi)$ $\therefore V=A\omega\cos(\omega t+\phi)$

$$\therefore KE=\tfrac{1}{2}mA^2\omega^2\cos^2(\omega t+\phi)$$

$$\therefore \boxed{KE=\tfrac{1}{2}kA^2\cos^2(\omega t+\phi)} \quad \text{—④} \qquad \therefore (KE \propto \cos^2\theta) \text{ w.r.t. time}$$

* PE of SHM:

During linear SHM, the restoring force acting on particle is

$$F=-kx \quad \text{—⑤} \quad \text{—}(k=\text{force constant})$$

The workdone for small displacement dx against restoring force F is given by

$$dw=F(-dx) \quad \text{—⑥} \quad \text{-- From ⑤ \& ⑥}$$

$$dw=-kx(-dx)$$

$$dw=kx\,dx \quad \text{—⑦}$$

The total workdone to displace particle from O to P is given by integrating eqⁿ ⑦ from limit O to x.

$$\therefore W=\int_0^x dw=\int_0^x kx\,dx$$

$$\therefore W=\tfrac{1}{2}kx^2 \quad \text{—⑧} \quad \text{---}(W=PE)$$

Eqⁿ ⑧ gives PE of particle at displacement x

$$\therefore \boxed{PE=\tfrac{1}{2}kx^2=\tfrac{1}{2}m\omega^2x^2} \quad \text{—⑨}$$

Also as $x=A\sin(\omega t+\phi)$

$$\therefore PE=\tfrac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$$

$$\boxed{PE=\tfrac{1}{2}kA^2\sin^2(\omega t+\phi)} \quad \text{—⑩} \quad \text{---}(PE \propto \sin^2\theta) \text{ w.r.t. time}$$

* **TE OF SHM:** TE of a particle is the sum of KE and PE.

$\therefore$ TE = KE + PE —— (11)

put (3) & (9) into (11)

$$TE = \frac{1}{2}m\omega^2(A^2-x^2) + \frac{1}{2}m\omega^2x^2$$

$$\boxed{TE = \frac{1}{2}m\omega^2A^2} \quad \text{———} \ (\omega = 2\pi n)$$

$$\boxed{TE = \frac{1}{2}kA^2} \quad \text{—— (12)}$$

$\because$ At amplitude A, velocity of particle is max^m (V_{max})

$$\therefore \boxed{TE = \frac{1}{2}mV_{max}^2} \quad \text{— (13)}$$

put $\omega = 2\pi n$ or $\omega = \frac{2\pi}{T}$ in eq^n (12)

$$\therefore \boxed{TE = 2\pi^2n^2A^2m}$$

$$\boxed{TE = 2\pi^2 m\frac{A^2}{T^2}} \quad \text{— (14)}$$

***Special cases:**

1) **At mean position:**

At mean position $x = 0$, KE is max^m & PE is zero

$\therefore KE = \frac{1}{2}m\omega^2A^2$

$PE = 0$ $\quad \therefore TE = \frac{1}{2}m\omega^2A^2$

2) **At extreme position:**

At extreme position $x = \pm A$, KE is zero & PE is max^m.

$\therefore KE = 0$

$PE = \frac{1}{2}m\omega^2A^2$ $\quad \therefore TE = \frac{1}{2}m\omega^2A^2$.

3) **If KE = PE**

$$\frac{1}{2}m\omega^2(A^2-x^2) = \frac{1}{2}m\omega^2x^2$$

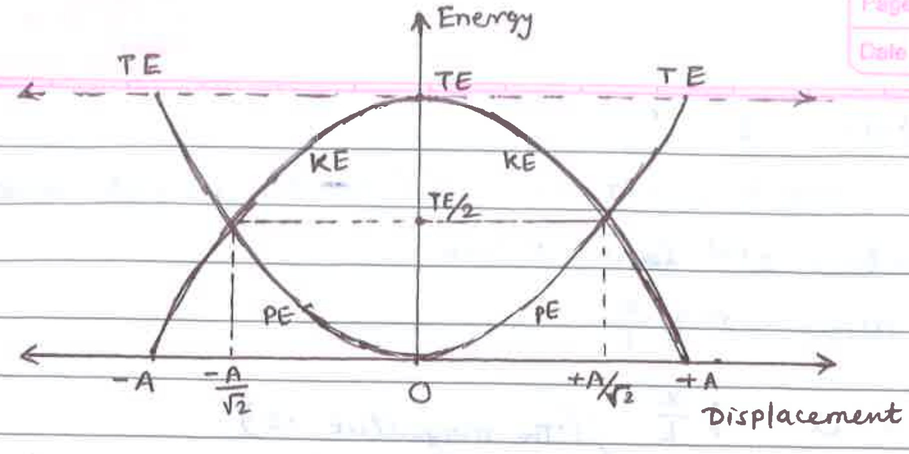
$\therefore x = \frac{\pm A}{\sqrt{2}}$ $\quad \therefore KE = PE = \frac{TE}{2}$.

4) **At $x = \frac{\pm A}{2}$**

$$PE = \frac{1}{2}kx^2 = \frac{1}{4}\left(\frac{1}{2}kA^2\right) = \frac{TE}{4}$$

$\therefore KE = 3PE$.

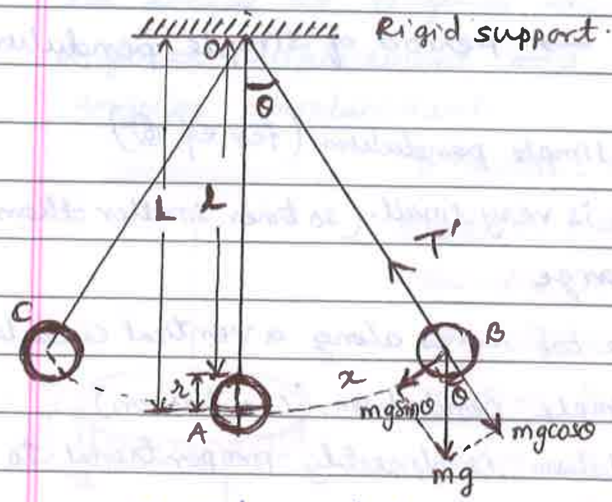
ie PE is 25% and KE is 75%



Que Define simple pendulum. Draw its neat diagram & obtain an expression for its period and frequency.

$\Rightarrow$ • **Ideal simple pendulum:-** It is a heavy particle (point mass) suspended by a massless, inextensible & flexible string from a rigid support.

• **Practical simple pendulum:-** It is a small heavy sphere (bob) suspended by a light & inextensible string from a rigid support.



m = mass of heavy sphere (bob)
l = Length of string
L = l + r = length of pendulum.
θ = Angular displacement
x = linear displacement
mg = weight of pendulum
T' = Tension in the string.

Consider a bob of mass 'm', suspended by a string of length l ($L = l + r$) from a rigid support. When pendulum is displaced through small angle θ & released, it begins to oscillate about mean position.

At extreme position B, the forces acting on the bob are.

1) Tension T' in the string towards point of support.
2) Weight mg in the vertically downward.

The force 'mg' is resolved into two components.

a) $mg\cos\theta$ - along the string and balances with T'.
b) $mg\sin\theta$ - $\perp$er to the string, acts as restoring force towards mean position.

$\therefore F = -mg\sin\theta$ — (1) $\quad$ - - - ($\theta < 10^\circ$, $\sin\theta = \theta$)

(-ve sign is due to F & θ are opposite).

$\therefore F = -mg\theta$ — (2)

From fig. $\theta = \frac{x}{L}$

$\therefore F = -mg\frac{x}{L}$ —③ --- ($F \propto x$, m, g & L = constant)

By newtons 2^{nd} law $F = ma$

$\therefore ma = -mg\frac{x}{L}$

$\therefore a = -g\frac{x}{L}$, (The magnitue is)

$\therefore \frac{a}{x} = \frac{g}{L}$ —④ (accⁿ per unit displacement)

The period of simple pendulum is

$T = \frac{2\pi}{\omega}$ ---- ($a = \omega^2 x \Rightarrow \omega = \sqrt{a/x}$)

$\therefore T = \frac{2\pi}{\sqrt{\text{acc}^n \text{ per unit displacement}}}$ —⑤

From eqⁿ ④ & ⑤

$\boxed{T = 2\pi\sqrt{\frac{L}{g}}}$ —⑥ $\rightarrow$ period of simple pendulum.

Assumption for period of simple pendulum (for eqⁿ ⑥)

1) The amplitude of oscillation is very small (20 times smaller than length)
2) The length of string is large
3) During the oscillations, the bob moves along a vertical circular plane.

From eqⁿ ⑥ – Laws of simple pendulum. (conclusion).

1) The period of simple pendulum is directly proportional to the square root of its length.
2) The period of simple pendulum is inversely proportional to the square root of accⁿ due to gravity.
3) The period of simple pendulum does not depends upon its mass.
4) The period of simple pendulum does not depends upon its amplitude (for small amplitude).

The freqⁿ of oscillation of simple pendulum is

$n = \frac{1}{T}$ —⑦

From eqⁿ ⑥ & ⑦

$\boxed{n = \frac{1}{2\pi}\sqrt{\frac{g}{L}}}$ —⑧

* Second's Pendulum :- A simple pendulum whose period is 2 seconds is called as second's pendulum.

The period of simple pendulum is

$T = 2\pi\sqrt{\frac{L}{g}}$ —①

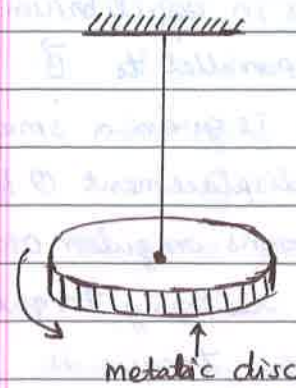
For second's pendulum $T = 2$ sec.

$2 = 2\pi\sqrt{\frac{L_s}{g}}$

$\therefore \boxed{L_s = \frac{g}{\pi^2}}$ —② $\rightarrow$ Length of second's pendulum.

Que – Define angular SHM. Obtain its differential equation & state the formula of its period.

$\Rightarrow$ • Angular SHM :- It is an oscillatory motion of a body in which the torque for angular accⁿ is directly proportional to the angular displacement and its direction is opposite to that of angular displacement.



A metalic disc attached centrally to a thin wire (nylon or metal) hanging from a rigid support. If the disc is slightly twisted about the axis along the string & released, it performs rotational motion partly in clockwise and anticlockwise sense. Such oscillations are angular or torsional oscillations.

For angular SHM, the torque acting on disc is

$\tau \propto -\theta$ --- (-ve sign is due to opposite direction of τ & θ).

$\therefore \tau = -C\theta$ —①

(where C = restoring torque per unit angular displacement = constant)

If I is MI of the body & α is angular accⁿ, the torque is

$\tau = I\alpha$ —② --- ($\alpha = \frac{d^2\theta}{dt^2}$)

$\therefore \tau = I\frac{d^2\theta}{dt^2}$ —②

From ① & ②

$I\frac{d^2\theta}{dt^2} = -C\theta$

$$\therefore \boxed{I\frac{d^2\theta}{dt^2} + C\theta = 0} \text{ — (3)}$$

eq^n (3) is differential eq^n of angular SHM.

Also $\alpha = \frac{d^2\theta}{dt^2} = -\frac{C\theta}{I} \Rightarrow \frac{d^2\theta/dt^2}{\theta} = -\frac{C}{I}$.

ie $\alpha \propto -\theta$, so this oscillatory motion is called angular SHM.

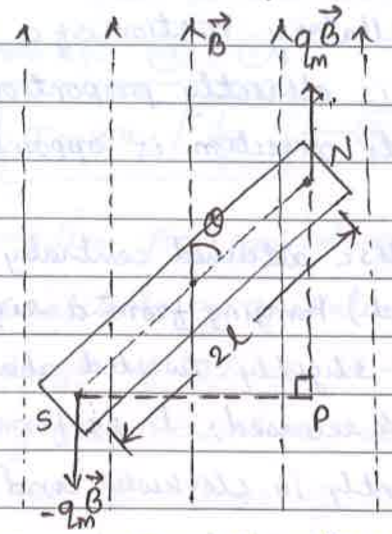
The period of angular SHM is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\text{Angular acc}^n\text{ per unit angular displacement}}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{C}}} \text{ — (4)}$$

Que - Describe the magnet vibrating in a uniform magnetic field and find the equation of its period.

⟹



A bar magnet of magnet length $2l$ & magnetic dipole moment μ is placed in uniform magnetic field $\vec{B}$. It remains in equilibrium with its axis parallel to $\vec{B}$.

If it is given a small angular displacement θ & released, it performs angular oscillations.

In the deflected position, a restoring torque acts on the magnet. The magnitude of this torque is

$\tau = \mu B \sin\theta$ — (1)

$\because \theta$ is very small $\therefore \sin\theta = \theta^\circ$.

$\therefore \tau = \mu B\theta$ — (2) --- (τ & θ are opposite)

but $\tau = I\alpha$ — (3)

$\therefore$ From (2) & (3)

$\alpha = \frac{\mu B}{I}\cdot\theta$ — (4) $\Rightarrow \frac{\alpha}{\theta} = \frac{\mu B}{I}$.

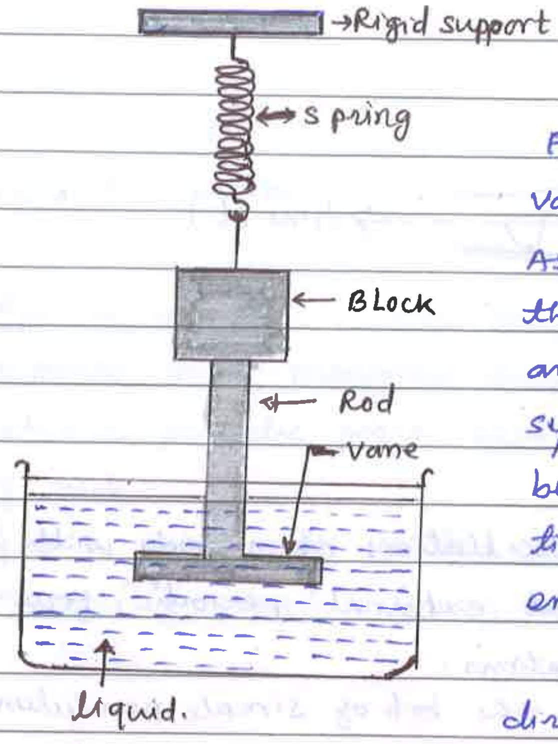
The period of magnet is

$$T = \frac{2\pi}{\sqrt{\text{Angular disp. per unit angular displacement}}}$$

$$\therefore \boxed{T = 2\pi\sqrt{\frac{I}{\mu B}}} \text{ — (5)}$$

Que - Define damped harmonic oscillator. With neat diagram find its differential equation and plot the graph of displacement against time.

⟹ Damped harmonic Oscillator :- The periodic oscillations of gradually decreasing amplitude are damped harmonic oscillations & the oscillator is called as damped harmonic oscillator.



A block of mass 'm' can oscillate vertically on a spring. From the block, a rod extends to vane that is submerged in a liquid. As the vane moves up and down, the liquid exerts drag force on it; and thus on the complete oscillating system. The mechanical energy of the block-spring system decreases with time & it is transfer to thermal energy of the liquid and vane. The damping force (F_d) is directly proportional to the speed v of the vane and the block.

$\therefore F_d = -bv$ — (1) --- (b = damping constant), (F_d is oppo. to v.)

The force on the block from the spring is

$F_s = -kx$ — (2) --- (k = spring constant)

Assume gravitational force on the block is negligible as F_d & F_s.

$\therefore$ Total force acting on the mass at time 't' is

$F = F_d + F_s$ — (3) --- ($\because F = ma$)

$\therefore ma = -bv - kx$

$ma + bv + kx = 0$ — (4)

but $a = \frac{d^2x}{dt^2}$, $v = \frac{dx}{dt}$

$$\therefore \boxed{m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0} \text{ — (5)} \rightarrow \text{Differential eq}^n\text{ of damped harmonic oscillator.}$$

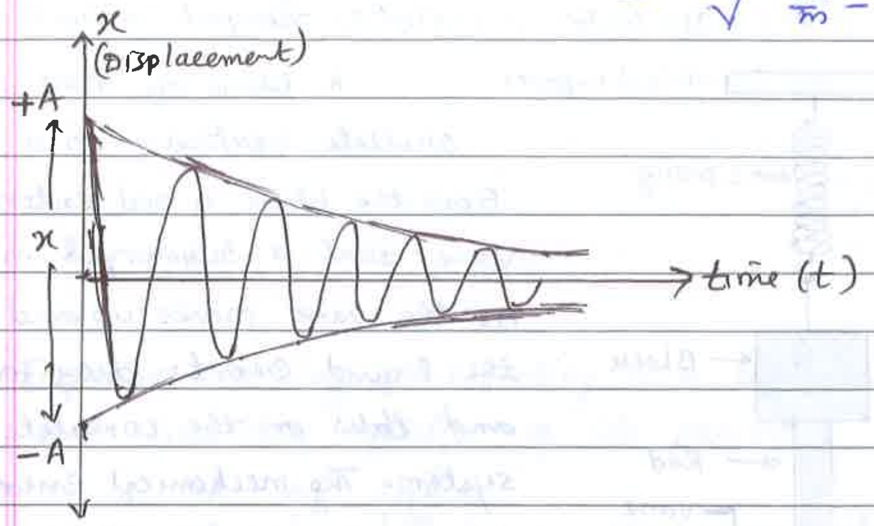
The solution of eq^n (5) is

$x = Ae^{-bt/2m}\cos(\omega' t + \phi)$ — (6) (where $Ae^{-bt/2m}$ is amplitude of oscillation)

The term $\cos(\omega' t + \phi)$ shows that the motion is an SHM.

The angular freq^n is $\omega' = \sqrt{\frac{k}{m} - \left(\frac{b^2}{2m}\right)}$

The period of Oscillation is $T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}}$.



* **Free Oscillations:-** The oscillations of a body with its natural freq^n without external periodic force are called as free oscillations.

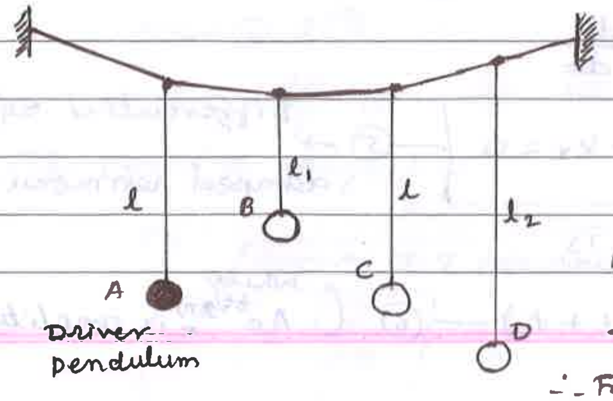
eq. 1) The oscillations of the bob of simple pendulum without external periodic force.

2) The ~~pro~~ oscillations of the prongs of tuning fork when struck on a rubber pad.

* **Forces Oscillations:** The Oscillations of a body with a freq^n of external periodic force other than its natural freq^n.

eq. 1) The vibrations of string with a vibrating tuning forke placed on a sonometer box.

2) The vibrations of an air column in a pipe of resonance tube due to vibrating tuning fork held above the open end of pipe.



A - Driver pendulum

B, C & D = Driven pendula

A = Solid ball (rubber)

B, C & D = hollow ball (rubber)

length of A = length of C.

∴ Freq^n of A = Freq^n of C.

Pendulum A & C are of equal lengths, while B is shorter & D is of longer length. $\left(n = \frac{1}{2\pi}\sqrt{g/l}\right)$. Due to equal lengths of A & C, they are of same natural frequency.

When A is set into vibration, through the string the energy is transfer to B, C & D. Pendulum C vibrate with max^m amplitude due to same freq^n as A. Freq^n of B is greater & that of D is less than A. So B & D are vibrate with some amplitude but less than C, due to forced vibrations.

* **Resonance:-** It is the phenomenon in which the body vibrate with maximum amplitude, when the freq^n of external periodic force becomes equal to the natural freq^n of body.

eq. 1) A paper rider on the string of sonometer will fall when the freq^n of vibrating tuning fork equals to the natural freq^n of string.